जब तक आपको यह परीक्षण पुस्तिका खोलने को न कहा जाए तब तक न खोलें

क्रम संख्या

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परीक्षण पुस्तिका अनुक्रम

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# परीक्षण पुस्तिका

## गणित

समय : दो घण्टे और तीस मिनट

पूर्णांक : 300

### अनुदेश

1. परीक्षा प्रारम्भ होने के तुरन्त बाद, आप इस परीक्षण पुस्तिका की पड़ताल अवश्य कर लें कि इसमें कोई बिना छपा, फटा या छूटा हुआ पृष्ठ अथवा प्रश्नांश, आदि न हो । यदि ऐसा है, तो इसे सही परीक्षण पुस्तिका से बदल लें ।
2. कृपया ध्यान रखें कि OMR उत्तर-पत्रक में, उचित स्थान पर, रोल नम्बर और परीक्षण पुस्तिका अनुक्रम (सीरीज कोड) A, B, C या D को, ध्यान से एवं बिना किसी चूक या विसंगति के भरने और कूटबद्ध करने की ज़िम्मेदारी उम्मीदवार की है । किसी भी प्रकार की चूक/विसंगति की स्थिति में उत्तर-पत्रक निरस्त कर दिया जाएगा ।
3. इस परीक्षण पुस्तिका पर साथ में दिए गए कोष्ठक में आपको अपना अनुक्रमांक लिखना है । परीक्षण पुस्तिका पर ***और कुछ न*** लिखें ।
4. इस परीक्षण पुस्तिका में 120 प्रश्नांश (प्रश्न) दिए गए हैं । प्रत्येक प्रश्नांश ***हिन्दी*** और ***अंग्रेज़ी*** दोनों में छपा है । प्रत्येक प्रश्नांश में चार प्रत्युत्तर (उत्तर) दिए गए हैं । इनमें से एक प्रत्युत्तर को चुन लें, जिसे आप उत्तर-पत्रक पर अंकित करना चाहते हैं । यदि आपको ऐसा लगे कि एक से अधिक प्रत्युत्तर सही हैं, तो उस प्रत्युत्तर को अंकित करें जो आपको सर्वोत्तम लगे । प्रत्येक प्रश्नांश के लिए ***केवल एक ही*** प्रत्युत्तर चुनना है ।
5. आपको अपने सभी प्रत्युत्तर अलग से दिए गए उत्तर-पत्रक पर ***ही*** अंकित करने हैं । उत्तर-पत्रक में दिए गए निर्देश देखें ।
6. ***सभी*** प्रश्नांशों के अंक समान हैं ।
7. इससे पहले कि आप परीक्षण पुस्तिका के विभिन्न प्रश्नांशों के प्रत्युत्तर उत्तर-पत्रक पर अंकित करना शुरू करें, आपको प्रवेश प्रमाण-पत्र के साथ प्रेषित अनुदेशों के अनुसार कुछ विवरण उत्तर-पत्रक में देने हैं ।
8. आप अपने सभी प्रत्युत्तरों को उत्तर-पत्रक में भरने के बाद तथा परीक्षा के समापन पर ***केवल उत्तर-पत्रक*** अधीक्षक को सौंप दें । आपको अपने साथ परीक्षण पुस्तिका ले जाने की अनुमति है ।
9. कच्चे काम के लिए पत्रक परीक्षण पुस्तिका के अंत में संलग्न हैं ।
10. **ग़लत उत्तरों के लिए दण्ड :**

    **वस्तुनिष्ठ प्रश्न-पत्रों में उम्मीदवार द्वारा दिए गए ग़लत उत्तरों के लिए दण्ड दिया जाएगा ।**

    (i) प्रत्येक प्रश्न के लिए चार वैकल्पिक उत्तर हैं । उम्मीदवार द्वारा प्रत्येक प्रश्न के लिए दिए गए एक ग़लत उत्तर के लिए प्रश्न हेतु नियत किए गए अंकों का **एक-तिहाई** दण्ड के रूप में काटा जाएगा ।

    (ii) यदि कोई उम्मीदवार एक से अधिक उत्तर देता है, तो इसे **ग़लत उत्तर** माना जाएगा, यद्यपि दिए गए उत्तरों में से एक उत्तर सही होता है, फिर भी उस प्रश्न के लिए उपर्युक्तानुसार ही उसी तरह का दण्ड दिया जाएगा ।

    (iii) यदि उम्मीदवार द्वारा कोई प्रश्न हल नहीं किया जाता है, अर्थात् उम्मीदवार द्वारा उत्तर नहीं दिया जाता है, तो उस प्रश्न के लिए **कोई दण्ड नहीं** दिया जाएगा ।

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Note : English version of the instructions is printed on the back cover of this Booklet.

1. The Cartesian product $A \times A$ has 16 elements among which are (0, 2) and (1, 3). Which of the following statements is/are correct ?

1. It is possible to determine set A.
2. $A \times A$ contains the element (3, 2).

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

2. Let $A = \{1, 2, 3, ..., 20\}$. Define a relation R from A to A by $R = \{(x, y) : 4x - 3y = 1\}$, where $x, y \in A$. Which of the following statements is/are correct ?

1. The domain of R is {1, 4, 7, 10, 13, 16}.
2. The range of R is {1, 5, 9, 13, 17}.
3. The range of R is equal to codomain of R.

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) 1 and 2

(d) 2 and 3

3. Consider the following statements :

1. The relation f defined by

$$f(x) = \begin{cases} x^3, & 0 \le x \le 2 \\ 4x, & 2 \le x \le 8 \end{cases}$$ is a function.

2. The relation g defined by

$$g(x) = \begin{cases} x^2, & 0 \le x \le 4 \\ 3x, & 4 \le x \le 8 \end{cases}$$ is a function.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

4. Consider the following statements :

1. $A = (A \cup B) \cup (A - B)$
2. $A \cup (B - A) = (A \cup B)$
3. $B = (A \cup B) - (A - B)$

Which of the statements given above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

5. A function satisfies $f(x-y) = \frac{f(x)}{f(y)}$, where $f(y) \neq 0$. If $f(1) = 0{\cdot}5$, then what is $f(2) + f(3) + f(4) + f(5) + f(6)$ equal to ?

(a) $\frac{15}{32}$

(b) $\frac{17}{32}$

(c) $\frac{29}{64}$

(d) $\frac{31}{64}$

6. What is $2\cot\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ equal to ?

(a) $-1$

(b) 1

(c) $3+\sqrt{5}$

(d) $3-\sqrt{5}$

7. If $\sec^{-1}p - \text{cosec}^{-1}q = 0$, where $p > 0$, $q > 0$; then what is the value of $p^{-2} + q^{-2}$ ?

(a) 1

(b) 2

(c) $\frac{1}{2}$

(d) $\frac{1}{2\sqrt{2}}$

8. What is $1 + \sin^2\left(\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)\right)$ equal to ?

(a) $\frac{25}{17}$

(b) $\frac{8}{17}$

(c) $\frac{9}{17}$

(d) $\frac{47}{17}$

9. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, $0 < \theta < \frac{\pi}{2}$; then what is the value of $8\sin^2\left(\theta + \frac{\pi}{4}\right)$ ?

(a) 16

(b) 2

(c) 1

(d) $\frac{1}{2}$

10. If $\tan\alpha = \frac{1}{7}$, $\sin\beta = \frac{1}{\sqrt{10}}$; $0 < \alpha, \beta < \frac{\pi}{2}$, then what is the value of $\cos(\alpha + 2\beta)$ ?

(a) $-\frac{1}{2}$

(b) $-\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{1}{2}$

***Consider the following for the next two (02) items that follow :***

Let $f(x) = x^2 - 1$ and $gof(x) = x - \sqrt{x} + 1$.

11. Which one of the following is a possible expression for g(x) ?

(a) $\sqrt{x+1} - \sqrt[4]{x+1}$

(b) $\sqrt{x+1} - \sqrt[4]{x+1} + 1$

(c) $\sqrt{x+1} + \sqrt[4]{x+1}$

(d) $x + 1 - \sqrt{x+1} + 1$

12. What is g(15) equal to ?

(a) 1

(b) 2

(c) 3

(d) 4

***Consider the following for the next two (02) items that follow :***

Let a function f be defined on $\mathbb{R} - \{0\}$ and $2f(x) + f\left(\frac{1}{x}\right) = x + 3$.

13. What is f(0·5) equal to ?

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) 1

(d) 2

14. If f is differentiable, then what is f′(0·5) equal to ?

(a) $\frac{1}{4}$

(b) $\frac{2}{3}$

(c) 2

(d) 4

***Consider the following for the next two (02) items that follow :***

A function is defined by

$$f(x) = \begin{vmatrix} x+1 & 2 & 3 \\ 2 & x+4 & 6 \\ 3 & 6 & x+9 \end{vmatrix}.$$

15. The function is decreasing on :

(a) $\left[-\frac{28}{3}, 0\right]$

(b) $\left[0, \frac{28}{3}\right]$

(c) $\left[0, \frac{50}{3}\right]$

(d) $\left[0, \frac{56}{3}\right]$

16. The function attains local minimum value at :

(a) $x = -\frac{28}{3}$

(b) $x = -1$

(c) $x = 0$

(d) $x = \frac{28}{3}$

***Consider the following for the next two (02) items that follow :***

Given that $4x^2 + y^2 = 9$.

17. What is the maximum value of y ?

(a) $\frac{3}{2}$

(b) 3

(c) 4

(d) 6

18. What is the maximum value of xy ?

(a) $\frac{9}{4}$

(b) $\frac{3}{2}$

(c) $\frac{4}{9}$

(d) $\frac{2}{3}$

***Consider the following for the next two (02) items that follow :***

A function is defined by $f(x) = \pi + \sin^2 x$.

19. What is the range of the function ?

(a) $[0, 1]$

(b) $[\pi, \pi + 1]$

(c) $[\pi - 1, \pi + 1]$

(d) $[\pi - 1, \pi - 1]$

20. What is the period of the function ?

(a) $2\pi$

(b) $\pi$

(c) $\frac{\pi}{2}$

(d) The function is non-periodic

***Consider the following for the next two (02) items that follow :***

Let $I = \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1 + 3^x} dx$

21. What is $\int_0^{\pi} (\sin^4 x + \cos^4 x)\, dx$ equal to ?

(a) $\frac{3\pi}{8}$

(b) $\frac{3\pi}{4}$

(c) $\frac{3\pi}{2}$

(d) $3\pi$

22. What is I equal to ?

(a) 0

(b) $\frac{3\pi}{4}$

(c) $\frac{3\pi}{2}$

(d) $3\pi$

***Consider the following for the next two (02) items that follow :***

Let $f(x) = \begin{cases} \frac{ax(x+1) + b}{x - 1}, & x < 1 \\ , & 1 \le x \le 2 \end{cases}$

23. If the function f(x) is differentiable at x = 1, then what is the value of (a + b) ?

(a) $\frac{1}{3}$

(b) $-1$

(c) 0

(d) 1

24. What is $\lim_{x \to 0} f(x)$ equal to ?

(a) $-\frac{1}{3}$

(b) $-\frac{2}{3}$

(c) 0

(d) 1

25. If $f(x) = |ln|x||$ where $0 < x < 1$, then what is $f'(0{\cdot}5)$ equal to ?

(a) $-2$

(b) $-1$

(c) 0

(d) 2

26. If $f'(x) = \cos(ln\ x)$ and $y = f\left(\frac{2x-3}{x}\right)$, then what is $\frac{dy}{dx}$ equal to ?

(a) $\cos\left(ln\left(\frac{2x-3}{x}\right)\right)$

(b) $-\frac{3}{x^2}\sin\left(ln\left(\frac{2x-3}{x}\right)\right)$

(c) $\frac{3}{x^2}\cos\left(ln\left(\frac{2x-3}{x}\right)\right)$

(d) $-\frac{3}{x^2}\cos\left(ln\left(\frac{2x-3}{x}\right)\right)$

27. What is $\int_0^{8\pi} |\sin x|\,dx$ equal to ?

(a) 2

(b) 4

(c) 8

(d) 16

28. What is the area between the curve $f(x) = x|x|$ and x-axis for $x \in [-1, 1]$ ?

(a) $\frac{2}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $\frac{1}{3}$

29. What are the order and the degree respectively of the differential equation $x^2\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + \sin x = 0$ ?

(a) 3, 4

(b) 1, 4

(c) 2, 2

(d) 3, 2

30. What is the differential equation of all parabolas of the type $y^2 = 4a(x - b)$ ?

(a) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(b) $\frac{d^2y}{dx^2} + x^2\left(\frac{dy}{dx}\right)^2 = 0$

(c) $y^2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(d) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

31. If $z\bar{z} = |z + \bar{z}|$, where $z = x + iy$, $i = \sqrt{-1}$, then the locus of z is a pair of :

(a) straight lines

(b) rectangular hyperbolas

(c) parabolas

(d) circles

32. If $1! + 3! + 5! + 7! + \ldots + 199!$ is divided by 24, what is the remainder ?

(a) 3

(b) 6

(c) 7

(d) 9

33. What is the value of $\sqrt{12 + 5i} + \sqrt{12 - 5i}$, where $i = \sqrt{-1}$ ?

(a) 24

(b) 25

(c) $5\sqrt{2}$

(d) $5(\sqrt{2} - 1)$

34. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then what is the value of $\det(I + AA')$, where I is the $3 \times 3$ identity matrix ?

(a) 15

(b) 6

(c) 0

(d) $-1$

35. If A, B and C are square matrices of order 3 and $\det(BC) = 2\det(A)$, then what is the value of $\det(2A^{-1}BC)$ ?

(a) 16

(b) 8

(c) 4

(d) 2

36. If the $n^{th}$ term of a sequence is $\frac{2n+5}{7}$, then what is the sum of its first 140 terms ?

(a) 2840

(b) 2780

(c) 2920

(d) 5700

37. Let A be a skew-symmetric matrix of order 3. What is the value of

$\det(4A^4) - \det(3A^3) + \det(2A^2) - \det(A) + \det(-I)$

where I is the identity matrix of order 3 ?

(a) $-1$

(b) 0

(c) 1

(d) 2

38. If $A = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$, then which one of the following statements is correct ?

(a) $A^2$ is symmetric matrix with $\det(A^2) = 0$.

(b) $A^2$ is symmetric matrix with $\det(A^2) \neq 0$.

(c) $A^2$ is skew-symmetric matrix with $\det(A^2) = 0$.

(d) $A^2$ is skew-symmetric matrix with $\det(A^2) \neq 0$.

39. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then which of the following statements are correct ?

1. $A^n$ will always be singular for any positive integer n.
2. $A^n$ will always be a diagonal matrix for any positive integer n.
3. $A^n$ will always be a symmetric matrix for any positive integer n.

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

40. If (a + b), 2b, (b + c) are in HP, then which one of the following is correct ?

(a) a, b and c are in AP

(b) a – b, b – c and c – a are in AP

(c) a, b and c are in GP

(d) a – b, b – c and c – a are in GP

***Consider the following for the next two (02) items that follow :***

Consider the equation $(1 - x)^4 + (5 - x)^4 = 82$.

41. What is the number of real roots of the equation ?

(a) 0

(b) 2

(c) 4

(d) 8

42. What is the sum of all the roots of the equation ?

(a) 24

(b) 12

(c) 10

(d) 6

***Consider the following for the next three (03) items that follow :***

Consider equation-I : $z^3 + 2z^2 + 2z + 1 = 0$ and equation-II : $z^{1985} + z^{100} + 1 = 0$.

43. What are the roots of equation-I ?

(a) $1, \omega, \omega^2$

(b) $-1, \omega, \omega^2$

(c) $1, -\omega, \omega^2$

(d) $-1, -\omega, -\omega^2$

44. Which one of the following is a root of equation-II ?

(a) $-1$

(b) $-\omega$

(c) $-\omega^2$

(d) $\omega$

45. What is the number of common roots of equation-I and equation-II ?

(a) 0

(b) 1

(c) 2

(d) 3

*Consider the following for the next two (02) items that follow :*

A quadratic equation is given by $(a+b)x^2 - (a+b+c)x + k = 0$, where a, b, c are real.

46. If $k = \frac{c}{2}$, $(c \neq 0)$, then the roots of the equation are :

(a) Real and equal

(b) Real and unequal

(c) Real iff $a > c$

(d) Complex but not real

47. If $k = c$, then the roots of the equation are :

(a) $\frac{a+c}{a+b}$ and $\frac{b}{a+b}$

(b) $\frac{a+c}{a+b}$ and $-\frac{b}{a+b}$

(c) 1 and $\frac{c}{a+b}$

(d) $-1$ and $-\frac{c}{a+b}$

***Consider the following for the next three (03) items that follow :***

Let $(1+x)^n = 1 + T_1x + T_2x^2 + T_3x^3 + \ldots + T_nx^n$.

48. What is $T_1 + 2T_2 + 3T_3 + \ldots + nT_n$ equal to ?

(a) 0

(b) 1

(c) $2^n$

(d) $n2^{n-1}$

49. What is $1 - T_1 + 2T_2 - 3T_3 + \ldots + (-1)^n nT_n$ equal to ?

(a) 0

(b) $-2^{n-1}$

(c) $n2^{n-1}$

(d) 1

50. What is $T_1 + T_2 + T_3 + \ldots + T_n$ equal to ?

(a) $2^n$

(b) $2^n - 1$

(c) $2^{n-1}$

(d) $2^n + 1$

51. If r is the coefficient of correlation between x and y, then what is the correlation coefficient between $(3x + 4)$ and $(-3y + 3)$ ?

(a) $-r$

(b) $r$

(c) $\sqrt{3}\,r$

(d) $-\sqrt{3}\,r$

52. A fair coin is tossed 6 times. What is the probability of getting a result in the $6^{th}$ toss which is different from those obtained in the first five tosses ?

(a) $\frac{7}{16}$

(b) $\frac{1}{16}$

(c) $\frac{1}{32}$

(d) $\frac{1}{64}$

53. If H is the Harmonic Mean of three numbers $^{10}C_4$, $^{10}C_5$, and $^{10}C_6$, then what is the value of $\frac{270}{H}$ ?

(a) 1

(b) $\frac{14}{17}$

(c) $\frac{17}{14}$

(d) $\frac{1}{31}$

54. In a class, there are n students including the students P and Q. What is the probability that P and Q sit together if seats are assigned randomly ?

(a) $\frac{1}{n}$

(b) $\frac{2}{n}$

(c) $\frac{4}{n}$

(d) $\frac{1}{2n}$

55. In a Binomial distribution B(n, p), n = 6 and $9P(X = 4) = P(X = 2)$. What is p equal to ?

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) $\frac{4}{5}$

*Consider the following for the next five (05) items that follow :*

Three boys P, Q, R and three girls S, T, U are to be arranged in a row for a group photograph.

56. What is the probability that all three boys sit together ?

(a) $\frac{1}{5}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $\frac{1}{12}$

57. What is the probability that boys and girls sit alternatively ?

(a) $\frac{4}{5}$

(b) $\frac{1}{10}$

(c) $\frac{5}{6}$

(d) $\frac{1}{7}$

58. What is the probability that no two girls sit together ?

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{1}{18}$

(d) $\frac{1}{5}$

59. What is the probability that P and Q take the two end positions ?

(a) $\frac{1}{15}$

(b) $\frac{7}{15}$

(c) $\frac{14}{15}$

(d) $\frac{11}{45}$

60. What is the probability that Q and U sit together ?

(a) $\frac{2}{3}$

(b) $\frac{1}{4}$

(c) $\frac{5}{6}$

(d) $\frac{1}{3}$

61. What is the length of projection of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$ on the vector $2\hat{i} + 3\hat{j} - 2\hat{k}$ ?

(a) $\frac{1}{\sqrt{17}}$

(b) $\frac{2}{\sqrt{17}}$

(c) $\frac{3}{\sqrt{17}}$

(d) $\frac{2}{\sqrt{14}}$

62. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{b}| = 4$, then what is the value of $|\vec{a}|$ ?

(a) 3

(b) 4

(c) 5

(d) 6

63. If $\theta$ is the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a} \cdot \vec{b} \geq 0$, then which one of the following is correct ?

(a) $0 \leq \theta \leq \pi$

(b) $\frac{\pi}{2} \leq \theta \leq \pi$

(c) $0 \leq \theta \leq \frac{\pi}{2}$

(d) $0 < \theta < \frac{\pi}{2}$

64. The vectors $60\hat{i} + 3\hat{j}$, $40\hat{i} - 8\hat{j}$ and $\beta\hat{i} - 52\hat{j}$ are collinear if :

(a) $\beta = 20$

(b) $\beta = 40$

(c) $\beta = -40$

(d) $\beta = 26$

65. Consider the following in respect of the vectors $\vec{a} = (0, 1, 1)$ and $\vec{b} = (1, 0, 1)$ :

1. The number of unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ is only one.

2. The angle between the vectors is $\frac{\pi}{3}$.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

66. If L is the line with direction ratios $< 3, -2, 6 >$ and passing through $(1, -1, 1)$, then what are the coordinates of the points on L whose distance from $(1, -1, 1)$ is 2 units ?

(a) $\left(-\frac{11}{7}, \frac{13}{7}, \frac{19}{7}\right)$ and $\left(\frac{1}{7}, \frac{3}{7}, \frac{5}{7}\right)$

(b) $\left(\frac{19}{7}, -\frac{11}{7}, \frac{13}{7}\right)$ and $\left(-\frac{1}{7}, \frac{3}{7}, -\frac{5}{7}\right)$

(c) $\left(\frac{13}{7}, \frac{11}{7}, \frac{19}{7}\right)$ and $\left(-\frac{1}{7}, -\frac{3}{7}, \frac{5}{7}\right)$

(d) $\left(\frac{13}{7}, -\frac{11}{7}, \frac{19}{7}\right)$ and $\left(\frac{1}{7}, -\frac{3}{7}, -\frac{5}{7}\right)$

67. Which one of the planes is parallel to the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ ?

(a) $2x + 2y + z - 1 = 0$

(b) $2x - y - 2z + 5 = 0$

(c) $2x + 2y - 2z + 1 = 0$

(d) $x - 2y + z - 1 = 0$

68. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ ?

(a) $0°$

(b) $30°$

(c) $60°$

(d) $90°$

69. What is the equation of the sphere concentric with the sphere $x^2 + y^2 + z^2 - 2x - 6y - 8z - 5 = 0$ and which passes through the origin ?

(a) $x^2 + y^2 + z^2 - 2x - 8z = 0$

(b) $x^2 + y^2 + z^2 - 2x - 6y = 0$

(c) $x^2 + y^2 + z^2 - 6y - 8z = 0$

(d) $x^2 + y^2 + z^2 - 2x - 6y - 8z = 0$

70. A point P lies on the line joining A(1, 2, 3) and B(2, 10, 1). If z-coordinate of P is 7, what is the sum of other two coordinates ?

(a) 15

(b) – 13

(c) – 11

(d) – 9

71. Let $t_1, t_2, t_3 \ldots$ be in GP. What is $(t_1 t_3 \ldots t_{21})^{\frac{1}{11}}$ equal to ?

(a) $t_{10}$

(b) $t_{10}^2$

(c) $t_{11}$

(d) $t_{11}^2$

72. Which one of the following is a square root of $-\sqrt{-1}$ ?

(a) $1 + i$

(b) $\frac{1-i}{\sqrt{2}}$

(c) $\frac{1+i}{\sqrt{2}}$

(d) $\frac{1}{\sqrt{2}} i$

73. What is the maximum number of points of intersection of 10 circles ?

(a) 45

(b) 60

(c) 90

(d) 120

74. A set S contains (2n + 1) elements. There are 4096 subsets of S which contain at most n elements. What is n equal to ?

(a) 5

(b) 6

(c) 7

(d) 8

75. If $\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$, then what is the value of e ?

(a) $-1$

(b) 0

(c) 1

(d) 2

76. If all elements of a third order determinant are equal to 1 or $-1$, then the value of the determinant is :

(a) 0 only

(b) an even number but not necessarily 0

(c) an odd number

(d) 0, 1 or $-1$

77. If $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then what is the value of $\det[\text{adj}(\text{adj}A)]$ ?

(a) 5

(b) 25

(c) 125

(d) 625

78. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then what is $23A^3 - 19A^2 - 4A$ equal to ?

(a) Null matrix of order 3

(b) Identity matrix of order 3

(c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

79. The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of $C^2$ ?

(a) 3

(b) 9

(c) 81

(d) 729

80. If $A_k = \begin{bmatrix} k-1 & k \\ k-2 & k+1 \end{bmatrix}$, then what is $\det(A_1) + \det(A_2) + \det(A_3) + \ldots + \det(A_{100})$ equal to ?

(a) 100

(b) 1000

(c) 9900

(d) 10000

***Consider the following for the next two (02) items that follow :***

A parabola passes through (1, 2) and satisfies the differential equation $\frac{dy}{dx} = \frac{2y}{x}$, $x > 0, y > 0$.

81. What is the directrix of the parabola ?

(a) $y = -\frac{1}{8}$

(b) $y = \frac{1}{8}$

(c) $x = -\frac{1}{8}$

(d) $x = \frac{1}{8}$

82. What is the length of latus rectum of the parabola ?

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $\frac{1}{8}$

***Consider the following for the next two (02) items that follow :***

Let $f(x) = \frac{a^{x-1} + b^{x-1}}{2}$ and $g(x) = x - 1$.

83. What is $\lim_{x \to 1} \frac{f(x) - 1}{g(x)}$ equal to ?

(a) $\frac{ln\,(ab)}{4}$

(b) $\frac{ln\,(ab)}{2}$

(c) $ln\,(ab)$

(d) $2\,ln\,(ab)$

84. What is $\lim_{x \to 1} f(x)^{g(x)}$ equal to ?

(a) $\sqrt{ab}$

(b) $ab$

(c) $2ab$

(d) $\frac{\sqrt{ab}}{2}$

***Consider the following for the next two (02) items that follow :***

Let $f(x) = \sqrt{2 - x} + \sqrt{2 + x}$.

85. What is the domain of the function ?

(a) $(-2, 2)$

(b) $[-2, 2]$

(c) $R - (-2, 2)$

(d) $R - [-2, 2]$

86. What is the greatest value of the function ?

(a) $\sqrt{3}$

(b) $\sqrt{6}$

(c) $\sqrt{8}$

(d) 4

***Consider the following for the next two (02) items that follow :***

Let $f(x) = |x|$ and $g(x) = [x] - 1$, where $[.]$ is the greatest integer function.

Let $h(x) = \frac{f(g(x))}{g(f(x))}$.

87. What is $\lim_{x \to 0+} h(x)$ equal to ?

(a) $-2$

(b) $-1$

(c) 0

(d) 1

88. What is $\lim_{x \to 0-} h(x)$ equal to ?

(a) $-2$

(b) $-1$

(c) 0

(d) 2

*Consider the following for the next two (02) items that follow :*

Let $f(x) = \begin{cases} \frac{x-3}{|x-3|} + a; & x < 3 \\ a - b; & x = 3 \text{ and} \\ \frac{x-3}{|x-3|} + b; & x > 3 \end{cases}$

f(x) be continuous at x = 3.

89. What is the value of a ?

(a) $-1$

(b) 1

(c) 2

(d) 3

90. What is the value of b ?

(a) $-1$

(b) 1

(c) 2

(d) 3

91. The sum of deviations of n numbers from 10 and 20 are p and q respectively. If $(p-q)^2 = 10000$, then what is the value of n ?

(a) 10

(b) 20

(c) 50

(d) 100

92. If $\bar{X} = 20$ is the mean of 10 observations $x_1, x_2, \ldots, x_{10}$; then what is the value of $\sum_{i=1}^{10} \left(\frac{3x_i - 4}{5}\right)$ ?

(a) 0

(b) 12

(c) 112

(d) 1012

93. If the mean and the sum of squares of 10 observations are 40 and 16160 respectively, then what is the standard deviation ?

(a) 16

(b) 6

(c) 5

(d) 4

94. Three dice are thrown. What is the probability of getting a sum which is a perfect square ?

(a) $\frac{17}{108}$

(b) $\frac{5}{108}$

(c) $\frac{19}{108}$

(d) $\frac{23}{108}$

95. A, B, C and D are mutually exclusive and exhaustive events.

If $2P(A) = 3P(B) = 4P(C) = 5P(D)$,

then what is $77P(A)$ equal to ?

(a) 12

(b) 15

(c) 20

(d) 30

96. Two distinct natural numbers from 1 to 9 are picked at random. What is the probability that their product has 1 in its unit place ?

(a) $\frac{1}{81}$

(b) $\frac{1}{72}$

(c) $\frac{1}{18}$

(d) $\frac{1}{36}$

97. Two dice are thrown. What is the probability that difference of numbers on them is 2 or 3 ?

(a) $\frac{7}{36}$

(b) $\frac{7}{18}$

(c) $\frac{5}{18}$

(d) $\frac{11}{36}$

98. What is the mean of the numbers 1, 2, 3, ..., 10 with frequencies ${}^9C_0$, ${}^9C_1$, ${}^9C_2$, ..., ${}^9C_9$, respectively ?

(a) $1{\cdot}1 \times 2^8$

(b) $1{\cdot}2 \times 7^4$

(c) 5·5

(d) 0·55

99. The probability that a person recovers from a disease is 0·8. What is the probability that exactly 2 persons out of 5 will recover from the disease ?

(a) 0·00512

(b) 0·02048

(c) 0·2048

(d) 0·0512

100. Suppose that there is a chance for a newly constructed building to collapse, whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that the building collapses is 95% if the design is faulty, otherwise it is 45%. If it is seen that the building has collapsed, then what is the probability that it is due to faulty design ?

(a) 0·10

(b) 0·19

(c) 0·45

(d) 0·95

101. What is the equation of directrix of parabola $y^2 = 4bx$, where $b < 0$ and $b^2 + b - 2 = 0$ ?

(a) $x + 1 = 0$

(b) $x - 2 = 0$

(c) $x - 1 = 0$

(d) $x + 2 = 0$

102. The points $(-a, -b)$, $(0, 0)$, $(a, b)$ and $(a^2, ab)$ are :

(a) lying on the same circle

(b) vertices of a square

(c) vertices of a parallelogram that is not a square

(d) collinear

103. Given that $16p^2 + 49q^2 - 4r^2 - 56pq = 0$. Which one of the following is a point on a pair of straight lines $(px + qy + r)(px + qy - r) = 0$ ?

(a) $\left(2, \frac{7}{2}\right)$

(b) $\left(2, -\frac{7}{2}\right)$

(c) $(4, -7)$

(d) $(4, 7)$

104. If $3x + y - 5 = 0$ is the equation of a chord of the circle $x^2 + y^2 - 25 = 0$, then what are the coordinates of the mid-point of the chord ?

(a) $\left(\frac{3}{4}, \frac{1}{4}\right)$

(b) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{3}{4}, -\frac{1}{4}\right)$

(d) $\left(\frac{3}{2}, -\frac{1}{2}\right)$

105. Consider the following in respect of the equation $\frac{x^2}{24-k} + \frac{y^2}{k-16} = 2$.

1. The equation represents an ellipse if $k = 19$.
2. The equation represents a hyperbola if $k = 12$.
3. The equation represents a circle if $k = 20$.

How many of the statements given above are correct ?

(a) Only one

(b) Only two

(c) All three

(d) None

106. Consider the following statements in respect of hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ :

1. The two foci are independent of $\theta$.
2. The eccentricity is sec $\theta$.
3. The distance between the two foci is 2 units.

How many of the statements given above are correct ?

(a) Only one

(b) Only two

(c) All three

(d) None

107. Consider the following in respect of the circle $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$ :

1. The circle touches both the axes.
2. The diameter of the circle is 2a.
3. The centre of the circle lies on the line $x + y = a$.

How many of the statements given above are correct ?

(a) Only one
(b) Only two
(c) All three
(d) None

108. For what values of k is the line $(k-3)x - (5-k^2)y + k^2 - 7k + 6 = 0$ parallel to the line $x + y = 1$ ?

(a) $-1, 1$
(b) $-1, 2$
(c) $1, -2$
(d) $2, -2$

109. The line $x + y = 4$ cuts the line joining P(–1, 1) and Q(5, 7) at R. What is PR : RQ equal to ?

(a) 1 : 1
(b) 1 : 2
(c) 2 : 1
(d) 1 : 3

110. What is the sum of the intercepts of the line whose perpendicular distance from origin is 4 units and the angle which the normal makes with positive direction of x-axis is 15° ?

(a) 8
(b) $4\sqrt{6}$
(c) $8\sqrt{6}$
(d) 16

***Consider the following for the next two (02) items that follow :***

Let $a_1, a_2, a_3 \ldots$ be in AP such that

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{25} + a_{30} + a_{34} = 300$.

111. What is

$a_1 + a_5 - a_{10} - a_{15} - a_{20} - a_{25} + a_{30} + a_{34}$

equal to ?

(a) 0
(b) 25
(c) 125
(d) 250

112. What is $\sum_{n=1}^{34} a_n$ equal to ?

(a) 900
(b) 1025
(c) 1200
(d) 1275

***Consider the following for the next two (02) items that follow :***

Let $p = \cos\left(\frac{\pi}{5}\right)\cos\left(\frac{2\pi}{5}\right)$ and $q = \cos\left(\frac{4\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right)$.

113. What is the value of p + q ?

(a) $-\frac{1}{2}$
(b) $-\frac{1}{4}$
(c) 0
(d) $\frac{1}{2}$

114. What is the value of pq ?

(a) $-\frac{1}{16}$

(b) $-\frac{1}{4}$

(c) $\frac{1}{4}$

(d) $\frac{1}{16}$

***Consider the following for the next two (02) items that follow :***

Let $p = \frac{1}{3} - \frac{\tan 3x}{\tan x}$ and

$q = 1 - 3\tan^2 x, 0 < x < \pi, x \neq \frac{\pi}{2}$.

115. What is pq equal to ?

(a) 1

(b) 2

(c) $\frac{8}{3}$

(d) $-\frac{8}{3}$

116. For how many values of x does $\frac{1}{p}$ become zero ?

(a) No value

(b) Only one value

(c) Only two values

(d) Only three values

***Consider the following for the next two (02) items that follow :***

Let $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$; $x + y = \frac{\pi}{2}$, $0 < x, y < \frac{\pi}{2}$.

117. What is a value of $\sin 3x + \sin 3y$ ?

(a) $-1$

(b) 0

(c) 1

(d) 3

118. What is a value of $\cos^3 x + \cos^3 y$ ?

(a) $\frac{3\sqrt{3}}{8}$

(b) $\frac{3\sqrt{6}}{8}$

(c) $\frac{3\sqrt{6}}{4}$

(d) 1

***Consider the following for the next two (02) items that follow :***

The angles A, B and C of a triangle ABC are in the ratio 3 : 5 : 4.

119. What is the value of $a + b + \sqrt{2}c$ equal to ?

(a) 3a

(b) 2b

(c) 3b

(d) 2c

120. What is the ratio of $a^2 : b^2 : c^2$ ?

(a) $2 : 2 + \sqrt{3} : 3$

(b) $2 : 2 - \sqrt{3} : 2$

(c) $2 : 2 + \sqrt{3} : 2$

(d) $2 : 2 - \sqrt{3} : 3$